# COMPUTER ALGORITHM FOR <br> <br> DIVERGENT-LIGHT <br> <br> DIVERGENT-LIGHT HALOS 

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# Why is are divergent-light halos difficult to simulate? 

Source

PLH


DLH


Parallel-light halos are just DIRECTIONAL.
Divergent-light halos are also SPATIAL

## Possible way of simulating

## Source



Observer

Very ineffective, most rays are wasted and don't enter the observer's eye.

Ray geometry


## THE ALGORITHM:

Assumptions:
(1) Intensity does not noticeably diminish due to scattering
(2) Neglect multiple scattering
(3) No diffraction

Power at the observer:

$$
P_{O}=\frac{A \sigma_{0} P n}{4 \pi} \int \frac{d^{3} \mathbf{a}}{a^{2} b^{2}} \int D \mathbf{U} Q(\mathbf{U}) p(\hat{\mathbf{b}} \mid \hat{\mathbf{a}}, \mathbf{U})
$$

$A=$ Observer's pupil area
$\sigma_{0}=$ Total scattering cross-section of a crystal
$P=$ Source intensity
$n=$ Number density of crystals
$\mathbf{U}=$ Crystal orientation
$Q(\mathbf{U})=$ Crystal orientation
distribution
$p(\hat{\mathbf{b}} \mid \hat{\mathbf{a}}, \mathbf{U})=$ Probability of scattering in direction $\hat{b}$ given incident direction â and orientation $\mathbf{U}$.

## Random orientation, isotropic scattering:

$$
P_{i s o}=\frac{A \sigma_{0} P n}{4 \pi} \int \frac{d^{3} \mathbf{a}}{a^{2} b^{2}} \int D \mathbf{U} 1 \cdot \frac{1}{4 \pi}=\frac{A \sigma_{0} P n \pi}{4 R}
$$

Normalised quantity:
$\mathrm{Z}=\frac{P_{\mathrm{O}}}{P_{\text {iso }}}$
or

$$
Z=\frac{R}{\pi^{2}} \int \frac{d^{3} \mathbf{a}}{a^{2} b^{2}} \int D \mathbf{U} Q(\mathbf{U}) p(\hat{\mathbf{b}} \mid \hat{\mathbf{a}}, \mathbf{U})
$$

## Look at rays that have a constant

 scattering angle $\omega$

## Such rays define a MINNAERT CIGAR

## TWO EXAMPLES:

$\omega=45^{\circ}$ CIGAR


## $\omega=150^{\circ}$ CIGAR



# CRYSTALS THAT SCATTER A DEFINITE ANGLE HAVE TO BE LOCATED ON A MINNAERT CIGAR 

## Introduce bipolar angles:



Minnaert cigar

## The bipolar angles uniquely define a point on the cigar

In terms of bipolar angles we have
$\mathbf{a}=\frac{R \sin (\omega-\theta)}{\sin \omega}(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$
In particular the integration element becomes extremely simple
$\frac{d^{3} \mathbf{a}}{a^{2} b^{2}}=\frac{1}{R} d \omega d \theta d \phi$
This suggests that we should to try and represent the events using bipolar angles.

## The standard event:



[^0]
## Using

1) Standard events
2) The rotation symmetry
3) Bipolar angle representation
we arrive at the following expression (after some mathematics)
$Z=\frac{1}{4 \pi} \int d^{2} \hat{\mathbf{a}}_{0} \int d^{2} \hat{\mathbf{b}}_{0} \quad p\left(\hat{\mathbf{b}}_{0} \mid \hat{\mathbf{a}}_{0}, \mathbf{1}\right) \cdot W$
i. e. a sum of weighted standard events where the weight $W$ is

$$
W=\frac{32 \omega}{\sin \omega} \int_{0}^{\omega} \frac{d \theta^{2 \pi}}{\omega} \int_{0}^{2 \pi} \frac{d \phi}{2 \pi} Q(\mathbf{U})
$$

## IN PRACTICE:

1) Generate a standard event with
a random incident ray $\hat{\mathbf{a}}_{0}$, raytrace throught the crystal,
get the scattered ray $\hat{\mathbf{b}}_{0}$ with probability $p\left(\hat{\mathbf{b}}_{0} \mid \hat{\mathbf{a}}_{0}, \mathbf{1}\right)$
2) Compute the scattering angle
$\omega=\arccos \left(\hat{\mathbf{a}}_{0} \cdot \hat{\mathbf{b}}_{0}\right)$
3) Choose the angle $\theta$ randomly in the interval $[0, \omega]$ and the angle $\phi$ randomly in $[0,2 \pi]$ This defines a point on the Minnaert cigar associated with the angle $\omega$.
4) Make the cigar axis point to the observer and compute the vectors $\mathbf{a}$ and $\mathbf{b}$.
5) Compute the transformation $\mathbf{U}$ that that will rotate $\hat{\mathbf{a}}_{0}$ and $\hat{\mathbf{b}}_{0}$ into $\mathbf{a}$ and $\mathbf{b}$. Apply the same transformation to the crystal orientation. Get the value of $\mathrm{Q}(\mathbf{U})$ associated with the crystal orientation $\mathbf{U}$.
6) Draw a point in the direction $-\hat{\mathbf{b}}$ from the observer with the found probability.
7) Reuse the event a number of times proportional to $\frac{\omega}{\sin \omega}$. This takes care of the first "cigar factor" in the weight $W$ (and saves a lot of raytracing).

The cigar factor goes to infinity as the scattering angle goes to $180^{\circ}$. However, this is not a problem since such Minnaert cigars have

1) Events concentrated directly behind the lamp and thus screened and not seen,
2) Or concentrated directly behind the observer and thus screened by him,
3) Or correspond to events very far away and physical crystal clouds are always limited in size thus such events are automatically cut off.

In practice we used a cut-off of $178^{\circ}$ for the scattering angle.

TOTAL SCATTERED POWER FOR
ISOTROPIC SCATTERING OF RANDOMLY ORIENTED CRYSTAL IN A CLOUD WITH LARGE BUT FINITE RADIUS $r$.

$$
P_{t o t}=\frac{P n \sigma_{0}}{4 \pi} \int \frac{d^{3} \mathbf{a}}{a^{2}}=P n \sigma_{0} r
$$

MEAN FREE PATH $d=\frac{1}{n \sigma_{0}}$

$$
\Rightarrow \frac{P_{t o t}}{P}=\frac{r}{d} \ll 1 \Rightarrow r \ll d
$$

# THIS IS CONSISTENT WITH OUR ASSUMPTIONS, OUR THEORY IS NOT VALID FOR INFINITE CLOUDS OF ICE CRYSTALS (SEE ASSUMPTIONS EARLIER) 

# NOTE THAT THE THEORY IS VERY GENERAL AND COULD BE <br> APPLIED TO ANY SCATTERING OF RAYS FROM OBJECTS <br> ILLUMINATED BY A DIVERGENT LIGHT SOURCE 

# IF THE LIGHT SOURCE IS VERY <br> DISTANT AND THE CLOUD OF CRYSTALS FINITE WE CAN SIMULATE THE SITUATION BY HAVING THE CIGARS TRUNCATED CLOSE TO THE OBSERVER 



# IN THIS LIMIT, WE RECOVER THE ORDINARY PARALLELLIGHT HALOS, A CHECK THAT OUR THEORY IS CONSISTENT 

## RESULTS

NOTE THAT THE SIMULATIONS ARE MADE FOR A LIGHT SOURCE THAT RADIATES ISOTROPICALLY.

MOST ARTIFICIAL LIGHT SOURCES HAVE SOME KIND OF SCREEN THAT CUTS OUT SOME RAY DIRECTIONS.

ORIENTED PLATES:
TIP ANGLE $1.0^{\circ}$, c/a RATIO 0.3

SINGLY ORIENTED COLUMNS:<br>TIP ANGLE $0.5^{\circ}$, c/a RATIO 2.0

## PARRY ORIENTED COLUMNS: <br> TIP ANGLES $1.5^{\circ}$, c/a RATIO 2.0

THE DISTRIBUTION OF TIP ANGLE IS A GAUSSIAN WITH ZERO MEAN AND A STANDARD DEVIATION EQUAL TO THE TIP ANGLE.

THE c/a RATIO IS DEFINED AS THE RATIO BETWEEN THE DISTANCE ACROSS A BASAL FACE OF THE CRYSTAL AND THE DISTANCE BETWEEN THE BASAL FACES.


## SINGLY ORIENTED COLUMN CRYSTALS



PARRY ORIENTED COLUMNS


## THE FULL ARTICLE IS PUBLISHED AS

# An improved algorithm for simulations of divergent-light halos <br> L. Gislén, J. O. Mattsson and B. Söderberg <br> Applied Optics 44:18, 3638-3645 (2005) <br> IF YOU HAVE DIFFICULTIES OBTAINING THE ARTICLE CONTACT ME <br> LarsG@thep.lu.se <br> ON MY HOMEPAGE <br> http:// www.thep.lu.se/ ~larsg/ <br> YOU WILL FIND THE C++ CODE OF THE <br> ALGORITHM (SEPARATE CODE FOR THE RAYTRACING PART IS NOT SUPPLIED). 

## ALSO SOME SIMULATED STEREO PICTURES OF DIVERGENT-LIGHT DISPLAYS.


[^0]:    SYMMETRY
    $p\left(\hat{\mathbf{b}}_{0} \mid \hat{\mathbf{a}}_{0}, 1\right)=p\left(\mathbf{U} \hat{\mathbf{b}}_{0} \mid \mathbf{U} \hat{\mathbf{a}}_{0}, \mathbf{U} \cdot 1\right)=p(\hat{\mathbf{b}} \mid \hat{\mathbf{a}}, \mathbf{U})$
    The scattering probability is invariant under any joint rotation of the incident and scattered rays and the crystal orientation.

